

12 SOLIDS IN THE SPACE

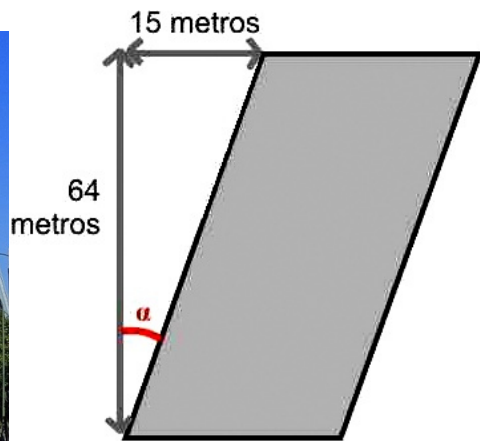
POLYHEDRONS AND SPACE

Everything has a place in space.

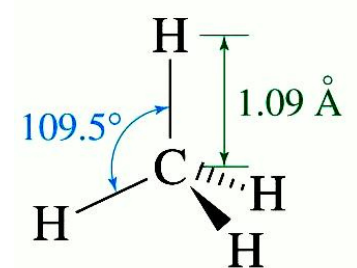
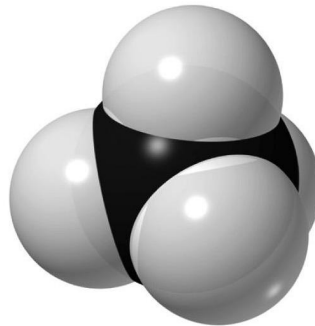
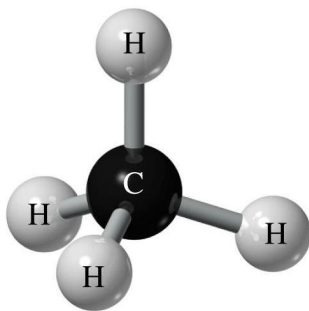
Buildings, structures, atoms, molecules... they take up¹ the space with a sense and according to its possibilities.

Here are some examples:

Kio towers



Methane atom



¹ Ocupar

Honeycomb



Sunflower

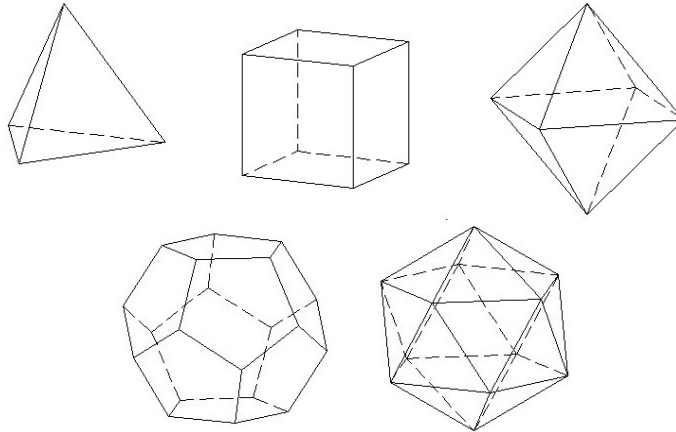


Sunflower seeds



STRAW POLYHEDRONS

These are the five regular polyhedrons, the platonic solids



Tools

In order to make straw polyhedrons you need:

1. Coloured non-bendable plastic straws wider than usual.
2. Thin cotton twine (plenty of it), and scissors to cut it.
3. A threading needle.

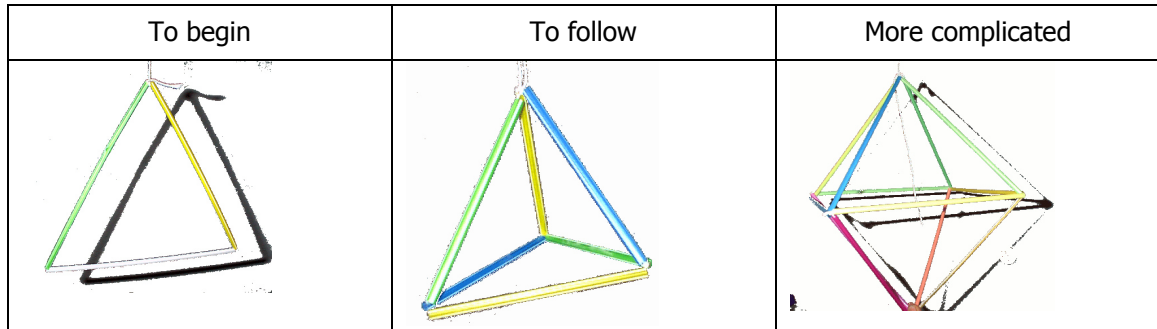


How to make it?

You have to use the needle and the twine to join the straws and to get the polyhedron.

You have to think of the best way to string all the straws to get the planned polyhedron.

Finally you can place the polyhedron hanging from the ceiling. Use different coloured straws according to your taste.

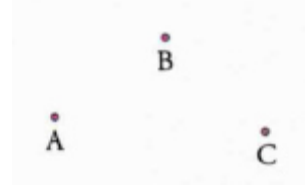


1. BASIC ELEMENTS IN SPACE

DEFINITIONS

Point

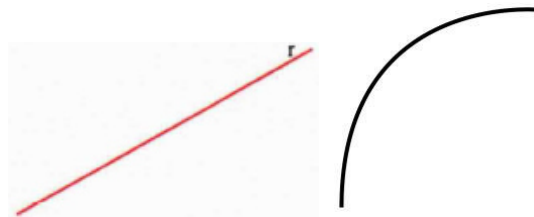
A point is the simplest object in the space. It has no size, only position.



Lines

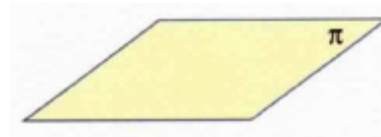
A line is a continuous sequence of points. The unit of measurement is the meter.

We have straight lines and curved lines.



Planes

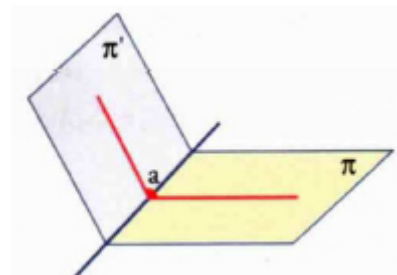
This object can be defined as the surface delimited by two straight lines with a common point.



ANGLES

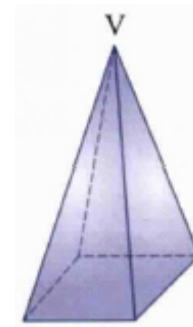
Dihedron angle

It is the separation between two planes with a common line. This is the rotation angle from one plane to another.



Polyhedron angle

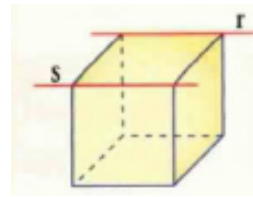
It is the sum of angles formed by consecutive faces with a common vertex.



POSITIONS FOR TWO STRAIGHT LINES IN SPACE

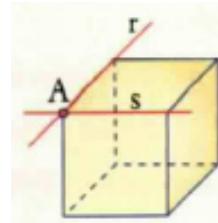
Parallels

Two straight lines which are in the same plane but without common point.



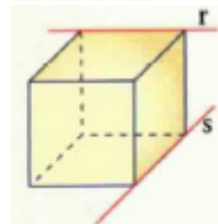
Secants

Two straight lines which are in the same plane but with a common point. They intersect at a point.



Cross

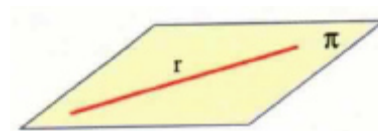
They don't have any common point and they are in different planes.



POSITIONS FOR A STRAIGHT LINE AND A PLANE

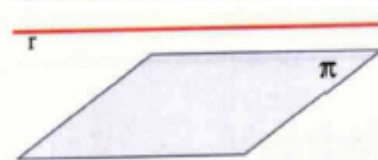
Straight line inside a plane

Every point is in the plane.



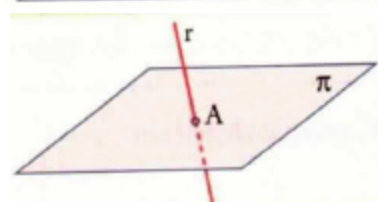
Parallels

A straight line is parallel to a plane if they don't have any common point.



Secants

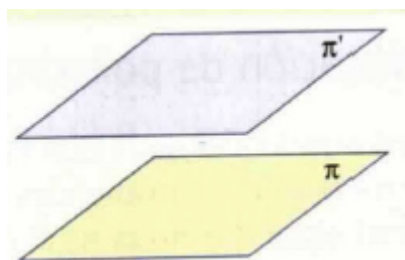
A straight line is secant to a plane if they have a common point.



POSITIONS FOR TWO PLANES IN SPACE

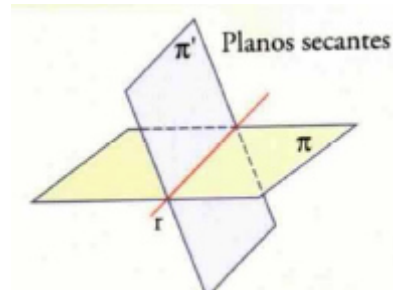
Parallels

Two planes are parallels if they don't have any common point.



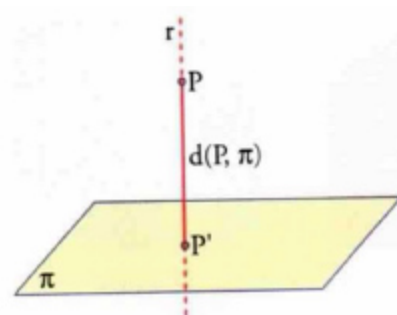
Secants

Two planes are secants if they have a common line.



Distance from one point to a plane

It is the shortest way from the point to the plane. It is the length between this point and the point determined in the plane by a perpendicular from the point to the plane.



2. POLYHEDRONS

DEFINITIONS

A polyhedron $[,pɒlɪ'hɪːdrən]$
 $N(p/\text{polyhedrons or polyhedra}) [,pɒlɪ'hɪːdrə]$ is the solid which is limited by polygons.

Face is each polygon.

Edge is each segment that is between two consecutive faces.

Vertex is each point that is common to many edges.

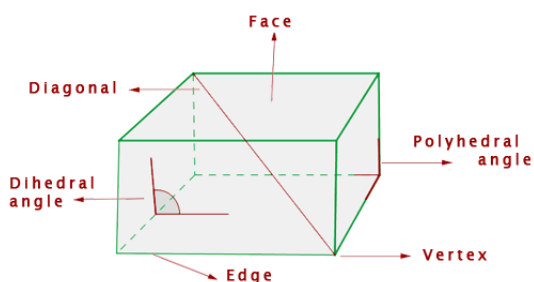
Euler's formula

It is verified that in all convex polyhedrons:

The number of faces plus the number of vertices equals the number of edges plus 2.

No. of faces + No. of vertices = No. of edges + 2.

$$F + V = E + 2$$



3. TYPES OF POLYHEDRA

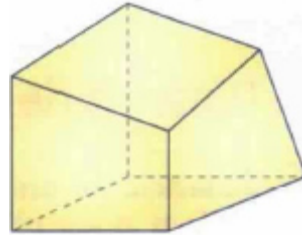
Regular

It is a polyhedron whose faces are regular polygons and equal all of them. So it has equal angles and faces. The platonic solids are convex regular polyhedrons.



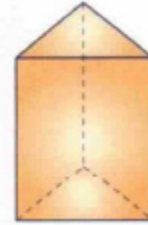
Irregular

It has irregular polygons as faces.



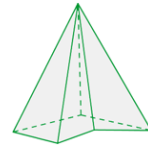
Convex

It is the polyhedron whose dihedral angles are less than 180° . The line segment joining any two points of the polyhedron is contained in the interior or in the face.



Concave

It is the polyhedron that has a dihedral angle higher than 180° .

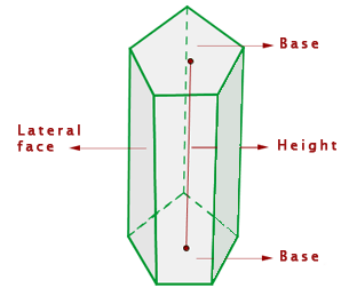


4. PRISMS AND CYLINDERS

PRISM

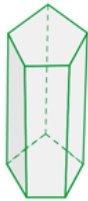
A prism ['prɪzəm] is a polyhedron with two parallel and equal faces named bases. The other faces are the parallelograms that join these faces. They are the lateral faces.

The height is the distance between the bases.



TYPES OF PRISMS

Regular Prisms



The bases of the regular prisms are regular polygons

Irregular Prisms



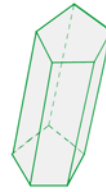
The bases of the irregular prisms are irregular polygons

Right Prisms



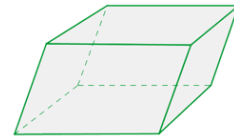
The lateral faces of the right prisms are rectangles or squares

Oblique Prisms



The lateral faces of the oblique prisms are rhomboids or rhombi.

Parallelepipeds [ˌpærəˈleləˈpiːpɪd]

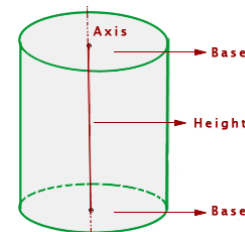


The bases of parallelepiped prisms are parallelograms

CYLINDERS

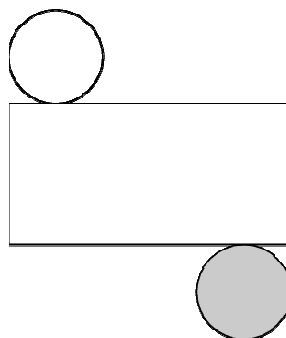
A curve solid is the solid which has some curve face.

The cylinder is the solid we get by rotating a rectangle around one of its faces.



UNFOLDING OF A CYLINDER

The unfolding of a right cylinder is formed by two circles -the bases- and a rectangle -the lateral surface-.



5. PYRAMIDS AND CONES

Definition

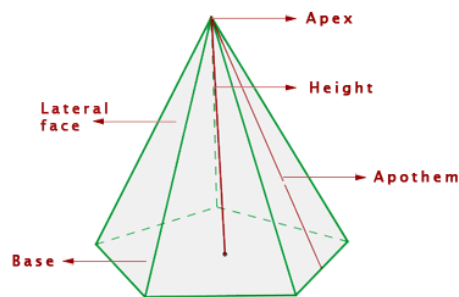
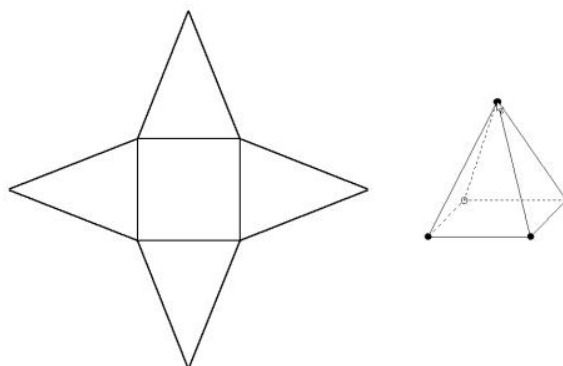
Pyramid is a polyhedron which faces are the following:

A polygon named base and the triangles formed joining the vertices with a fixed point named the vertex or apex ['eɪpeks] N(p/apexes, apices) ['eɪpɪsɪz] of the pyramid.

The height is the distance between the vertex and the base.

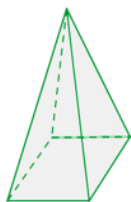
Unfolding of a Pyramid

The unfolding of a regular pyramid is formed by a regular polygon -the base- and as many isosceles and congruent triangles as lateral faces.



Types of Pyramids

Regular Pyramid



The base of a regular pyramid is a regular polygon and its lateral faces are equal.

Irregular Pyramid



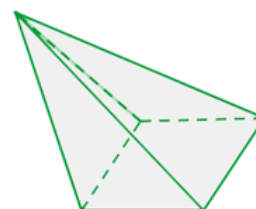
The base is an irregular polygon, and as a result, its faces are not equal.

Right Pyramid



it has isosceles triangles as its lateral faces and its apex lies directly above the midpoint of the base.

Oblique [ə'blɪk] Pyramid



An oblique pyramid does not have all isosceles triangles as its faces.

CONES

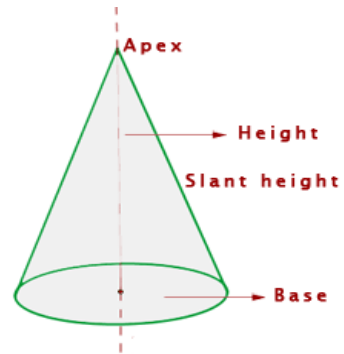
Definitions

A right cone is a solid of revolution generated by a right triangle that rotates around one of its legs.

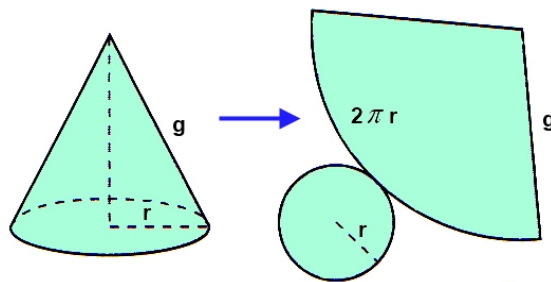
The height is the distance between the vertex and the base.

Generatrix is the segment from the vertex to a circle point.

In a right cone there is a Pythagorean relation:
 $G^2 = R^2 + H^2$



Unfolding of a right cone



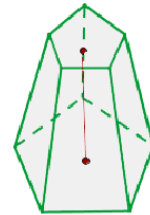
PYRAMID FRUSTUM OR TRUNCATED PYRAMID

Pyramid frustum ('frʌstəm) is the solid we get by cutting a pyramid by a parallel plane to the base. The lateral faces are trapezoids.

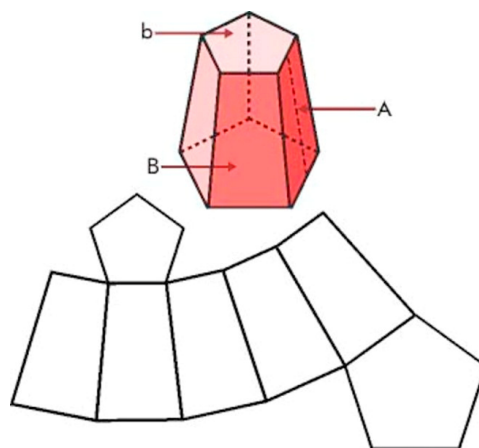
There are two important segments for the regular frustums:

The height or distance between the bases.

The apothem or height of each isosceles trapezium that it has as lateral faces.



Unfolding of a frustum



CONICAL FRUSTUM

Conical frustum is the solid we get by cutting a cone by a parallel plane to the base.

It is a solid of revolution generated by a right trapezium rotating around its height.

There are two important segments for the right conical frustum:

The height or distance between the bases.

The generatrix (d3enə, re1triks)

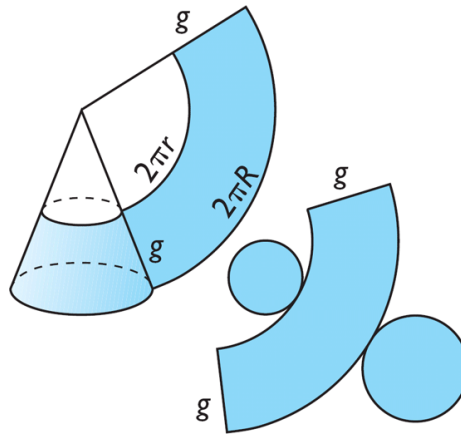
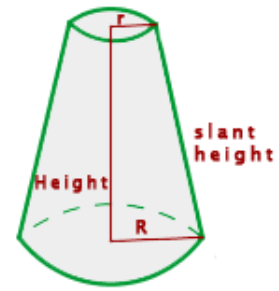
n, p /generatrices (d3enə, re1tri, siz)

or slant height from one base to the other.

In a right conical frustum there is a Pythagorean relation:

$$G^2 = (R-r)^2 + H^2$$

Unfolding of a Truncated Cone



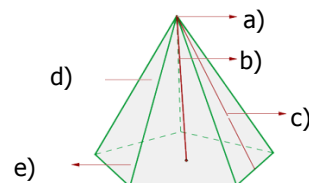
PROBLEMS AND EXERCISES

1. BASIC ELEMENTS IN THE SPACE

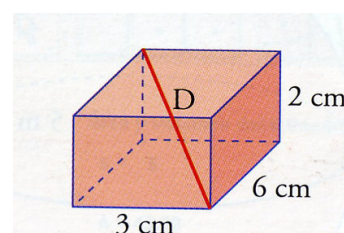
2. POLYHEDRON

1. Complete the gaps in the figure with a suitable word.

- a)
- b)
- c)
- d)
- e)

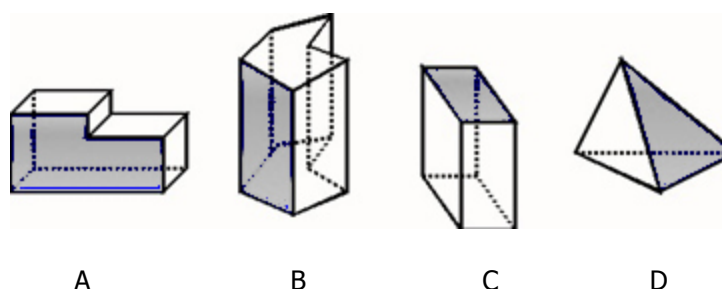


2. Work out the diagonal of the cuboid, the volume and the surface knowing that the length is 3 cm, the width is 6 cm and the height is 2 cm.



3. TYPES OF POLYHEDRA

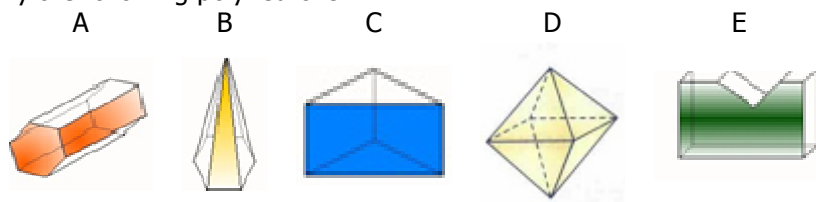
3. Classify the following polyhedrons:



- a) You have to write: concave or convex; regular or irregular.
- b) What kind of polyhedron are B, C and D respectively?
- c) Complete the table and verify Euler's formula?

	Nº of edges	Nº of faces	Nº of vertices	Euler's formula
A				
B				
C				
D				

4. Classify the following polyhedrons:



- a. What are concave polyhedrons?
- b. What kind of polyhedrons are according to the number of faces? (only A to D)
- c. What kind of polyhedrons are according to their shapes? (only A to D)
- d. Check the Euler's formula for A, B and C.

4. PRISMS AND CYLINDERS

5. PYRAMIDS AND CONES